posits in the cooling chamber), °K; θ, temperature difference between the central and boundary layers of the furnace-gas flow in the region of the outlet window, °K. Indices: i, j, r, zone numbers; dif, diffusional; nondif, nondiffusional; is, isotropic; anis, anisotropic; D, diffracted; inc, incident; G. Gas.

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REGULARIZATION IN THE PROBLEM OF DETERMINING EXTERNAL HEAT-TRANSFER CONDITIONS

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UDC 536.24:517.946

Questions are considered of the accuracy in determining the heat-elimination coefficient and the temperature of the environment by using the method of regularization according to the scheme of partial matching with elements of a set of observations.

A characteristic feature of many experimental investigations is the complexity of executing direct measurements of the desired quantities. Among such cases, for instance, is the known problem of determining the heat-transfer coefficient. Taking account of factors of nonstationarity of the process, nonlinearity of the thermophysical properties, and the spatial distribution of the heat complicates the application of traditional methods [1-3]. In this connection, methods based on the solution of the inverse heat-conduction problem are used to find the conditions for external heat exchange by means of measuring the temperature within the specimen in [4-7].

The problem of reproducing the cause according to the consequence being observed occurs constantly when studying the broadest class of phenomena. The isolation of inverse heat-conduction problems into a separate group and the development of a theory for identification of thermal processes [8, 9] is associated firstly with the complexity of obtaining final computational formulas since we only have available knowledge of certain model relationships implicitly expressing the connection between the temperature being observed and the parameters to be determined.

On the basis of the fact that the temperature field is a result of the properties of the test object and the conditions of its interaction with the environment, such model parameters

40th October Anniversary Balashikhinsk Scientific-Production Combine of Cryogenic Machine Construction.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 5, pp. 801-809, May, 1983. Original article submitted December 22, 1981.

0022-0841/83/4405-0548$07.50

are sought in inverse problems for which the temperature being observed and computed would be similar. In this case simplification of the model to obtain a computational dependence is not required and any known heat-transfer equations can be used that adequately describe the process being investigated.

Therefore, known methods of determining the thermophysical parameters by carrying over the subject of investigation from explicit to implicit functional dependences between the desired and given quantities are substantially extended to the solutions of inverse problems. These functional dependences can be expressed by differential equations for which it is difficult not only to obtain a final computational dependence but also an explicit analytical representation of the operator mapping the desired quantities on the set of initial data.

Henceforth, assuming the conditions satisfied for conservation of the mutual one-to-one correspondence between the heat-transfer coefficient and the temperature of the environment on the one hand, and the temperature field on the other [10], an investigation of questions of the stability and accuracy of determining the parameters of the thermal models by the method of regularization [11] by the scheme of partial matching with elements of the set of observations [12] was continued in this paper. Let us note that other Tikhonov regularization schemes were proposed earlier in [13, 14] for the case under consideration when the operator of the inverse problem has no explicitly analytic representation.

Let us set up the problem of simultaneously determining the heat-transfer coefficient and the environment temperature by means of the boundary heat-transfer conditions for a rod with a heat-insulated lateral surface. In this case the following boundary-value problem

$$
\begin{gathered}
c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial u}{\partial x}\right) + f(x,\ t),\ 0 < x < 1,\ 0 < t < T \\
u|_{t=0} = u_0(x), \quad 0 < x < 1; \\
\alpha\,(u|_{x=0} - u_{av}) - \lambda \left.\frac{\partial u}{\partial x}\right|_{x=0} = 0,\ 0 < t < T; \\
\alpha\,(u|_{x=1} - u_{av}) + \lambda \left.\frac{\partial u}{\partial x}\right|_{x=1} = 0,\ 0 < t < T,
\end{gathered}
\tag{1}
$$

can be the model, where the specific heat c, the density ρ, the heat conduction λ, the volume source intensity f(x, t), and the initial distribution $u_0(x)$ are considered given, while the parameters $\alpha(t)$ and $u_{av}(t)$ are to be determined.

Two kinds of formulations of the inverse problems are used to take simultaneous account of the observations. In one it is required to give an additional boundary condition in the boundary-value problem, while in the other the discrete set of observations on the solution that satisfies the model selected is considered known. One of the possible modifications of the second formulation is used below. It is formulated as follows.

It is required to find the vector $\bar{a} = \{\bar{\alpha},\ \bar{u}_{av}\}$ relative to which it is known that given observations u^δ containing the measurement interference ε have the prototypes $\bar{u}$, $u^\delta = \bar{u} + \varepsilon$ that satisfy the model (1) when its heat-transfer parameters are the desired quantities $\alpha = \bar{\alpha}$, $u_{av} = \bar{u}_{av}$. Relative to the experimental observations on the temperature field the sample $u^\delta = \{u^\delta_{ij}\}^{j=\overline{1,n}}_{i=\overline{1,m}}$ is considered given at m points of space and n times, which assures the identifiability of the desired heat-transfer conditions and has known estimates of the absolute measurement error $\delta = \{\delta_i\}_{i=\overline{1,m}}$ at each point of observation $\{x_i\}_{i=\overline{1,m}}$. It is also assumed that the selected model adequately describes the process under investigation, while the initial data permits finding a unique and stable solution of the direct problem. In particular, this latter condition requires a priori assignment of the appropriate properties of the functions $\alpha(t)$ and $u_{av}(t)$, for instance, their continuity. In the cases considered below these properties are considered known but the absence of any other additional information in the form of monotoneity, convexity, sections, extremum points and their like is assumed, which could indicate beforehand the exact nature of the desired dependences. This expands the domain of allowable solutions of the inverse problem and results in less favorable conditions from the viewpoint of accuracy of the solution. It should be noted that such an expansion is characteristic for many inverse problems.

To solve the inverse problem formulated, we use the following regularization scheme

$$\min \int_0^T \left[\left(\frac{d^p \alpha}{dt^p} \right)^2 + \left(\frac{d^q u_{av}}{dt^q} \right)^2 \right] dt;$$
$$\max_{1 \leqslant j \leqslant n} |u_{ij}^\delta - u_{ij}| \leqslant \delta_i, \quad i = \overline{1, m}, \tag{2}$$

where u is the temperature field computed by means of given values of the model parameters, p and q are the orders of the stabilizing functional in each of the desired quantities, δ_i is the absolute measurement error, m is the number of points of observation, and T is the upper bound of the observation time.

The regularization method is selected in connection with the fact that it permits taking complete account of the main singularities in the inverse problems, which are expressed in the possibility of broadening the domain of admissible solutions and in the appearance of an instability in the solution. Let us note that the proposed regularization differs from approaches that have been utilized extensively in inverse boundary-value problems [4, 8, 13]. In the formulation (2), only the $\Omega_{p,q}$ of the smoothing Tikhonov functional is minimized analogously to [11], but matching to the errors of the initial data is performed for each individual element of the set of observations. Consequently, there is no known scalar regularization parameter in (2) [11], and a requirement to obtain it in a certain set of measurement points $\{x_i\}_{i=\overline{1,m}}$ is imposed on the sample of observations. That last condition also expresses the distinction between a regularization scheme of the form (2) and other known variational schemes for solving incorrect problems [15, 16]. The separation of the samples into statistically independent groups required in (2) is, as is shown in [12, 17], an essential factor in the improvement of the accuracy of solving the inverse problem. Let us note that this question does not occur for the identification of one quantity, for instance, the boundary heat flux when knowledge of the observations at one point is sufficient. The passage to the identification of several parameters requires the knowledge of an appropriate number of additional observations. In this connection, the question of partial matching contains not only the singularity in the observations actually obtained, but also touches upon a general formulation of the problem.

Turning to the numerical realization of the solution of the problem under consideration, we reduce it to a mathematical programming problem. To do this we approximate the desired functions by using cubic splines [18]:

$$\alpha^{(l)}(t) = \sum_{i=1}^{4} s_{2l-2+i} P_i^{(l)}(t), \quad \tau_{l-1} \leqslant t \leqslant \tau_l;$$
$$u_{av}^{(l)}(t) = \sum_{i=1}^{4} s_{2l+2N_1+i} Q_i^{(l)}(t), \quad \vartheta_{l-1} \leqslant t \leqslant \vartheta_l, \tag{3}$$

where $\{s_i\}_{i=\overline{1,2(N_1+N_2+2)}}$ are coefficients to be determined, $P_i^{(l)}$ and $Q_i^{(l)}$ are second and third power polynomials at the approximation nodes $\{\tau_l\}_{l=\overline{0,N_1}}$ and $\{\vartheta_l\}_{l=\overline{0,N_2}}$; $N_{1,2}$ are parameters of the spline lattices setting up the number of approximation nodes.

Local properties of the splines permit a satisfactory description of a broad class of dependences. The general requirement for their utilization is smoothness of the function being represented. The desired coefficients $\{s_i\}$ for the splines are values of the function and its derivative at the approximation nodes. Use of the cubic splines allows evaluation of the stabilizing functionals $\Omega_{p,q}$ to third-order inclusive ($p, q = \overline{0.3}$).

To solve the mathematical programming problem related to determining the coefficients of the approximation $\{s_i\}$, we use the penalty method. It consists of minimizing the function

$$F(s) = \sum_{l=1}^{N_1} \int_{\tau_{l-1}}^{\tau_l} \left[\frac{d^p \alpha^{(l)}}{dt^p} \right]^2 dt + \sum_{l=1}^{N_2} \int_{\vartheta_{l-1}}^{\vartheta_l} \left[\frac{d^q u_{av}^{(l)}}{dt^q} \right]^2 dt + \sum_{i=1}^{m} K_i \gamma_i(s),$$

where $\gamma_i = \max_{1 \leqslant j \leqslant n} |u_{ij}^{\delta} - u_{ij}| - \delta_i$ is the residual in the matching conditions, K_i are the penalty coefficients ($K_i \gg 0$, if $\gamma_i < 0$ but $K_i = 0$ if $\gamma_i \leqslant 0$).

We shall seek the function u(x, t) satisfying the direct problem (1) for given coefficients $\{s_i\}$ by a finite-difference method with subsequent Bessel interpolation at intermediate values of the mesh function found.

On the basis of the above-mentioned method for determining the external heat-transfer conditions, we solve the following model problem. We assume the thermophysical properties of the rod to equal $c\rho = 1$, $\lambda = 1$, the initial distribution to be $u_0 = 100$, and the internal heat source has the power $f(x, t) = x$. Following [10], in this case it can be proved that the heat-transfer parameters $\alpha(t)$ and $u_{av}(t)$ in the model (1) are identifiable in the large and the uniqueness of their determination is allowed.

Let the desired quantities be described by the following functional dependences: varying slightly in time

$$\bar{\alpha} = 300 \exp\left(-\frac{t}{100}\right), \quad \bar{u}_{av} = 10 - \left(\frac{t}{T}\right)^5, \tag{4}$$

growing monotonically

$$\bar{\alpha} = 300 \sin\frac{\pi t}{2T}, \quad \bar{u}_{av} = 5 \operatorname{arctg} 10t \tag{5}$$

and unimodal

$$\bar{\alpha} = 300 - (7t - 3)^4, \quad \bar{u}_{av} = 2t \exp(-2t) + 1. \tag{6}$$

Using these values, we realize modeling of the observations according to the following law:

$$u_{ij}^{\delta} = \bar{u}(x_i, t_j) + \varepsilon_{ij}, \quad i = \overline{1, m}, \ j = \overline{1, n}, \tag{7}$$

where $\bar{u}$ is the solution of the direct problem (1) for which values of $\bar{\alpha}$ and $\bar{u}_{av}$ are given, and ε is the interference of the observations.

Measurement errors, inaccuracies in mounting the thermocouples, errors in modeling, and a number of other factors can be included in solving practical problems of the interference ε. Taking them all into account considerably complicates the analysis of the properties of the solution of the inverse problem. Spending the main attention on methodological questions of selecting the stabilizing functionals, we restrict the assignment of the observation interference to the form of white noise and system error. We determined values of the absolute measurement error from the formula

$$\delta_i = \max_{1 \leqslant j \leqslant n} |u_{ij}^{\delta} - \bar{u}_{ij}|, \quad i = \overline{1, m}.$$

Executing the modeling of the observations, we find the heat-transfer parameters by considering them smooth functions. The results of restoring the desired dependences of the form (4)-(6) by using the splines (3) are represented in the table, and some of them are shown in the figure. The upper bound of the observation time was taken at T = 1. The measurement interference ε was given with a normal distribution law and variance to which a relative measurement error of up to 3% corresponds. The sampling parameters had the value m = 2, n = 10. The approximation segment $0 \leq t \leq T$ was divided into five parts ($N_{1,2} = 5$), which assured approximation of the functions $\bar{\alpha}$ and $\bar{u}_{av}$ to an absolute accuracy not worse than 10^{-1}. The finite-difference approximation of the thermal model to obtain observations (7) and solutions of the inverse problem does not vary, i.e., modeling errors are not taken into account.

The results of the solution show that as the order of the stabilizer grows to the value p, q = 3, a continuous improvement occurs in the accuracy of the identification. The zeroth order of the stabilizer (p, q = 0) in the case of the approximation under consideration by cubic splines here turns out to be inadequate to a satisfactory solution of the problem formulated. Using $\Omega_{0,0}$ results in significant errors in identification despite the stability of the solution and execution of the matching conditions in the observations $\gamma_i < 0$. In combination with the results of preceding investigations [12, 17], the results obtained indicate a tendency to magnification of the constraints to a degree governed by the order of the highest stabilizer of the selected functional representation for the desired quantities. This

TABLE 1. Solution of Inverse Problems in Observations with Up to 3% Relative Error of Measurements

Form of desired quantities	Error in solution	Model (1)				Model (8)			
		$\Omega_{0,0}$	$\Omega_{1,1}$	$\Omega_{2,2}$	$\Omega_{3,3}$	$\Omega_{0,0}$	$\Omega_{1,1}$	$\Omega_{2,2}$	$\Omega_{3,3}$
(4)	$\int_0^T (\bar{\alpha}-\alpha)^2 dt$	12957	0,864	$3,82\times10^{-4}$	$8,58\times10^{-8}$	3370	0,765	$1,5\times10^{-3}$	$1,02\times10^{-6}$
	$\int_0^T (\bar{u}_{av}-u_{av})^2 dt$	1,262	0,064	0,0267	$3,37\times10^{-5}$	103,4	0,068	0,0238	$7,18\times10^{-5}$
(5)	$\int_0^T (\bar{\alpha}-\alpha)^2 dt$	28713	9374	558,3	0,1026	714,6	59,88	5,24	0,047
	$\int_0^T (\bar{u}_{av}-u_{av})^2 dt$	5,907	0,536	0,339	$2,37\times10^{-3}$	17,38	1,029	0,018	$1,23\times10^{-3}$
(6)	$\int_0^T (\bar{\alpha}-\alpha)^2 dt$	7468	3473	917,7	60,72	637,2	410,8	70,97	3,56
	$\int_0^T (\bar{u}_{av}-u_{av})^2 dt$	1,29	0,031	0,037	$2,1\times10^{-3}$	2,062	0,011	0,569	0,206

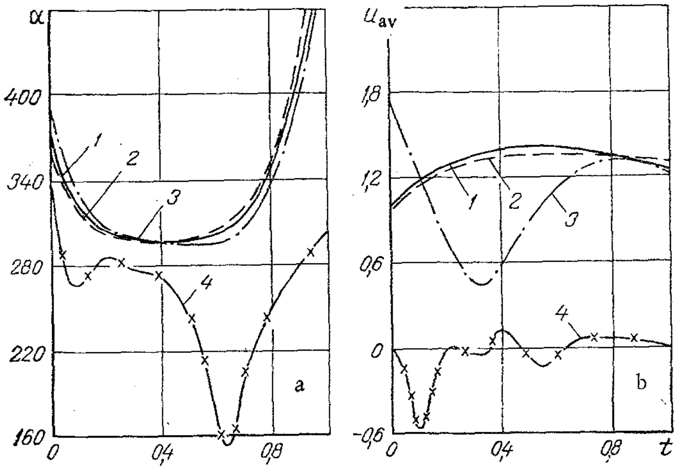

Fig. 1. Determination of the heat transfer (a) and ambient temperature (b) of the form (6): 1) exact value; 2) the model (8) $\Omega_{3.3}$, 3) the model (1) $\Omega_{3.3}$, 4) the model (1), $\Omega_{0.0}$ (a); 1) exact value; 2) the model (1) $\Omega_{3.3}$, 3) the model (8), $\Omega_{3.3}$, 4) the model (8) $\Omega_{0.0}$.

deduction shows the importance of exerting a sufficient degree of restraint on the domain of the allowable solutions. In the case of cubic splines such a constraint can be obtained by a stabilizing functional of third degree, which is related to the general properties of splines [18].

Comparing the results of determining the heat-transfer conditions of different nature, we turn attention to the growth of the error in identification in the restoration of dependences with a more complex nature.

Of indubitable interest is a study of the properties of solutions of the inverse problem when the level of the measurement interference increases. Let us execute an appropriate investigation by restoring a dependence of the form (6) and selecting the functional $\Omega_{3.3}$ among the stabilizers. Since the latter can later be recommended for the solution of practical problems, we present its final form in the case of approximating the desired quantities by cubic splines:

$$\Omega_{3,3}=\sum_{l=1}^{N_1}\frac{36}{\tau_l^3}\left(s_{2l-1}\frac{2}{\tau_l}+s_{2l}-s_{2l+1}\frac{2}{\tau_l}+s_{2l+2}\right)^2+$$

$$+\sum_{l=1}^{N_2}\frac{36}{\vartheta_l^3}\left(s_{2N_1+2l+1}\frac{2}{\vartheta_l}+s_{2N_1+2l+2}-s_{2N_1+2l+3}\frac{2}{\vartheta_l}+s_{2N_1+2l+4}\right)^2.$$

Processing the results of an experiment which were modeled by (7), where ε is interference with a normal distribution law and zero mathematical expectation, indicates satisfactory behavior of the solution of the inverse problem in the formulation (2) with the rise of the interference variance in this case. If the relative measurement errors are 50%, the identification error is 4% for the heat transfer and 30% for the ambient temperature. These results show that the error in solving the inverse problem in the formulation (2) does not exceed the level of the measurement errors when the stabilizing functional is selected appropriately.

In addition to modeling interference in the form of white noise, the presence of system error must be taken into account. To study this question, we give the interference in (7) in the form of a displacement in the quantity ε from the true temperature field $\bar{u}(x, t)$. In this case the solution of the inverse problem had an error to 3% for the heat transfer and 15% for the ambient temperature upon achievement of a system error of the level of 10% of the running value. Let us note that in contrast to the interference with a normal distribution law, the assignment of a system error in the case considered increases the growth in the error of identification of the function $u_{av}(t)$ with respect to the interference level.

Now, let us examine the properties of regularization by the scheme (2) when the desired external heat-transfer parameters enter the heat-conduction equation. Let the following model be considered

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial u}{\partial x}\right) - \alpha (u - u_{av}),\ 0 < x < 1,\ 0 < t < T;$$
$$u|_{t=0} = u_0(x),\ 0 < x < 1; \qquad (8)$$
$$u|_{x=0} = \varphi_0(t),\ u|_{x=1} = \varphi_1(t),\ 0 < t < T.$$

We verify the proposed method of determining the heat transfer and ambient temperature by the method elucidated above, by considering the thermophysical properties to depend on the temperature ($c\rho = 1 + u$, $\lambda = 1 + u$), the initial distribution and the boundary conditions are constant ($u_0 = 100$, $\varphi_{0,1} = 100$), while the desired quantities are described by dependences of the form (4)-(6).

Errors in the solutions are presented in the table as a function of the degree of constraint of the domain of allowable solutions. The results obtained in this case also indicate the necessity to magnify the constraints imposed on the domain of allowable solutions. The behavior of the solution as the variance in the measurement interference grows is analogous to the case considered above.

Therefore, it is shown that for the simultaneous determination of the heat-transfer coefficient and the ambient temperature, the application of the regularization method is effective. Here the selection of an appropriate degree of constraint on the domain of allowable solutions of the problem under consideration is quite important. The results obtained indicate a tendency to magnification of the constraints to a degree governed by the order of the highest stabilizer of the chosen functional representation of the desired quantities. Utilization of the regularization scheme with partial matching by elements of the set of observations permits execution of satisfactory identification in cases of increasing the variance of the interference. The practical value of the results obtained is in the development of a method to analyze the external heat transfer conditions by means of observations on the temperature within the body.

NOTATION

x, a space coordinate; t, time; u(x, t), temperature field; $\bar{u}$, true state of the process under investigation; u^δ, observation sample; δ, error in measurement; $\varphi_{0,1}$, boundary temperatures; α, heat-transfer coefficient; u_{av}, ambient temperature; $\Omega_{p,q}$, stabilizing functional; p and q, its order; and F, penalty function.

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SOLUTION OF THE HYPERBOLIC HEAT-CONDUCTION EQUATION BY EXPANSION IN A SMALL PARAMETER

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UDC 536.24.02

A method of finding the solution of the hyperbolic heat-conduction equation as a power series of a small parameter (the relaxation time) is discussed.

In the hyperbolic heat-conduction equation [1]

$$\frac{\partial T}{\partial \tau} + \tau_r \frac{\partial^2 T}{\partial \tau^2} = a \frac{\partial^2 T}{\partial x^2} \tag{1}$$

the relaxation time τ_r is small. For example in aluminum $\tau_r = 10^{-11}$ sec. Hence, one can consider (1) as an equation of a small parameter $\varepsilon = \tau_r/\tau_0$ and use asymptotic methods for its analysis and solution [2, 3].

We consider (1) (written in dimensionless form) for the following initial and boundary conditions:

$$T(x,\ 0) = \theta_0(x),\ \frac{\partial T(x,\ 0)}{\partial \tau} = \theta_1(x), \tag{2}$$

$$\beta_{i1} \frac{\partial T((i-1)l,\ \tau)}{\partial x} + (-1)^i \beta_{i2} T((i-1)l,\ \tau) = \varphi_i(\tau),\ i = 1,\ 2, \tag{3}$$

where depending on the type of boundary condition, the constants β_{i1}, β_{i2} are either equal to zero or correspond to the appropriate thermal constants.

Because (1) has the small parameter ε as a coefficient of the higher-order derivative, a power series expansion of the solution in ε must contain boundary-layer type terms depend-

Scientific-Industrial Union, "Tekhenergokhimprom," Mineral Fertilizer Industry, Moscow.
Translated from Inzhernerno-Fizicheskii Zhurnal, Vol. 44, No. 5, pp. 809-814, May, 1983. Original article submitted January 26, 1982.

